FINDING EQUATIONS OF LINES

- Want some practice with related concepts first?
  Introduction to Equations and Inequalities in Two Variables
  Introduction to the Slope of a Line
  Practice with Slope
  Graphing Lines

(more mathematical cats)

Every non-vertical line in the coordinate plane can be described by an equation of the form $y = mx + b$, where:

- m is the slope of the line
- b is where the line crosses the y-axis

The equation $y = mx + b$ is called the ***slope-intercept form of the line***.

Two different points uniquely determine a line.
One point and a slope also uniquely determine a line.
This web exercise gives you practice writing the equation of the line in these two situations.

EXAMPLE (KNOWN POINT, KNOWN SLOPE)

Question:
Find the equation of the line with slope 3 that passes through the point $(-1, 5)$.
Write the equation in $y = mx + b$ form.
Solution:

$y = mx + b$	(A line with slope 3 isn't vertical, so it can be described by an equation of this form.)
$y = 3x + b$	(Substitute the known slope, 3, in for m. Next, we must find b.)
$5 = 3(-1) + b$	(Since $(-1, 5)$ lies on the line, substitution of -1 for x and 5 for y makes the equation true.)
$5 = -3 + b$	(simplify)
$b = 8$	(add 3 to both sides; write in the conventional way)
$y = 3x + 8$	(substitute the now-known value of b into the equation)

Thus, the line with slope 3 that passes through $(-1, 5)$ is described by the equation $y = 3x + 8$.

Make sure you understand what this means!
Let ℓ denote the line with slope 3 that passes through the point $(-1, 5)$.
Every point that ***lies on*** ℓ has coordinates that make the equation $y = 3x + 8$ ***true***.
Every point that ***doesn't lie on*** ℓ has coordinates that make the equation $y = 3x + 8$ ***false***.

Head up to wolframalpha.com and type in:
`y = 3x + 8, x = -1, y = 5`
(Cut-and-paste, if you want.)
You'll see a graph of the line, with the given point indicated by crosshairs.
By adding in an additional set of crosshairs,
you can see that going up 3 and to the right 1 brings you to another point on the line:
`y = 3x + 8, x = -1, y = 5, x = 0, y = 8`

EXAMPLE (TWO KNOWN POINTS)

Question:
Find the equation of the line through the points $(2, -5)$ and $(-1, 4)$.
Write the equation in $y = mx + b$ form.
Solution:
First, use the slope formula to compute the slope:

$$\text{slope} = \frac{4 - (-5)}{-1 - 2} = \frac{9}{-3} = -3$$

Then, continue as in the previous example:

$y = mx + b$	(start with slope-intercept form)
$y = -3x + b$	(substitute the now-known slope, -3, in for m)
$4 = -3(-1) + b$	(Which point should you use? It doesn't matter! In general, try to choose the simplest numbers to work with.)
$4 = 3 + b$	(simplify)
$b = 1$	(subtract 3 from both sides; write in the conventional way)
$y = -3x + 1$	(substitute the now-known value of b into the equation)

You might want to check that the two points do indeed lie on the line:
$-5 \stackrel{?}{=} -3(2) + 1$ Check!
$4 \stackrel{?}{=} -3(-1) + 1$ Check!